Dr.V.S.KRISHNA GOVT.DEGREE COLLEGE (A), VISAKHAPATNAM.

COURSE-VII A

(w.e.f. 2022-23)

B.A./B.Sc. MATHEMATICS

CBCS/ SEMESTER SYSTEM

MATHEMATICAL SPECIAL FUNCTIONS

MODEL PAPER

Time:3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks5 X 5 M=25 M

1. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}}\, dx$.
2. Prove that $(1-x^2)T'_n(x) = -nxT'(x) + nT_{n-1}(x)$.
3. Determine the radius of convergence and the exact interval of convergence of each of the following power series
   $i)\ \sum \frac{nx^n}{(n+1)^2}$ $ii) \sum \frac{3^n x^n}{n!}$ $iii) \sum \frac{x^n}{n^3}$ $iv) \sum \frac{x^n}{n^n}$.
4. *Prove that* $H'_n(x) = 2n H_{n-1}(x), n \geq 1$.
5. Prove that $i)\ H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ and $ii)\ H_{2n+1}(0) = 0$.
6. (Beltrami's result) : $(2n+1)(x^2-1)P'_n = n(n+1)(P_{n+1} - P_{n-1})$.
7. Prove that i) $P'_n(1) = \frac{1}{2}n(n+1)$ $ii)\ P'_n(-1)^{n-1}\frac{1}{2}n(n+1)$.
8. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = x^{-n}J_{n+1}(x)$.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9. (a) Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

   OR

   (b) Show that $\frac{1}{\sqrt{(1-x^2}} U'_n(x)$ satisfies the differential equation

10. (a) Find the salutation in series of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$ about $x = 0$.

    OR

    (b) Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$.

11. (a) Solve by power series method $y'' - xy' = e^{-x}, y(0) = 2, y'(0) = -3$.

    OR

    (b)*Prove that* $\int_{-\infty}^{\infty} e^{-x^2} H_n(x)H_m(x)\, dx = \begin{cases} 0 & if\ m \neq n \\ \sqrt{\pi}\, 2^n\, n! & if\ m = n \end{cases}$

12. (a) Show that $(1-2hx+x^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x)$.

    OR

    (b) Prove that $\int_{-1}^{+1}(1-x^2)(P'_n)^2\, dx = \frac{2n(n+1)}{2n+1}$.

13. (a) Show that

i) $J_{-1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \cos x$, ii) $J_{1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$ and iii) $[J_{\frac{1}{2}}(x)] + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$.

OR

(b) Show that $x^n J^n(x)$ is solution of $x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0$.